BACKPAPER EXAMINATION
B. MATH II YEAR, I SEMESTER 2010-2011
ANALYSIS III

Max. 100. Time limit: 3hrs

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1. Find all sets $S \subset \mathbb{R}$ such that the series $\sum_{n=1}^{\infty} e^{nx}$ is uniformly convergent on S. Justify your answer. [15]

2. Prove or disprove: functions of the type $a_0 + a_1 x^2 + a_2 x^4 + ... + a_n x^{2n}$ $(n \geq 1, a_i's \in \mathbb{R})$ form a dense subset of $C[-2, 1]$.

($C[-2, 1]$ is the set of all continuous functions : $[-2, 1] \to \mathbb{R}$ with the metric d defined by $d(f, g) = \sup\{|f(x) - g(x)| : -2 \leq x \leq 1\}$]. [15]

3. Let $u(x, y) = (x^2 - y^2)e^x \cos y - 2xye^x \sin y$. Does there exist a function $v(x, y)$ such that $udx + vdy$ is exact? If so, find one such v. [15]

4. Define $d(x, y) = \max\{|x_i - y_i| : 1 \leq i \leq n\}$ for all $x, y \in \mathbb{R}^n$. Prove that a set A is open in $(\mathbb{R}^n, d)$ if and only if it is open in the usual metric. [10]

5. Let $f(x, y) = (y^2 + ye^x, 2xy - e^y)$. Does there exist a differentiable function $\phi : \mathbb{R}^2 \to \mathbb{R}$ such that $f = \nabla\phi$?. If so, find one such function. Otherwise prove that there is no such ϕ. [10]

6. Evaluate the surface integral of the function $F(x, y, z) = (xy, yz, x + z)$ over the surface $x^2 + y^2 + z^2 = 1, 0 \leq z \leq 1$ using DivergenceTheorem. [15]

7. Evaluate $\int_\Gamma (x^2 + y^2)^{99} 2xdx + (x^2 + y^2)^{99} 2ydy$ where Γ is the circle with center $(0, 0)$ and radius 1 using Green's Theorem. [5]

8. Find the volume of the solid inside the cylinder $x^2 + y^2 - 4y = 0$ lying between the plane $z = 0$ and the plane $z = 1$. [15]